

# Prefect Transfer of Quantum States on Spin Chain with Dzyaloshinskii- Moriya interaction in inhomogeneous Magnetic field

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## Abstract

In this work, we use the Hamiltonian of a modified Dzyaloshinskii-Moriya model and investigate the perfect transfer of the quantum state on the spin networks. In this paper, we calculate fidelity in which fidelity depends on magnetic field and another parameters. Then, by using the numerical analysis we show that the fidelity of the transferred state is determined by magnetic field  $B$ , exchange coupling  $J$  and the Dzyaloshinskii- Moriya interaction  $D$ . We also found that the perfect transfer of the quantum state is possible with condition  $B \gg \Gamma^2 \omega^{N/2}$  where  $\Gamma = ((J + iD)/2)$  and  $\omega = \Gamma^*/\Gamma$ .

**Keywords:** *XY* Spin-Chain, Teleportation, Dzyaloshinskii-Moriya interaction.

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# 1 Introduction

Quantum communication is the act of transferring a quantum state from one place to another. By far, its most well known application is quantum key distribution through which a secret random key can be established between distant parties with its security guaranteed by quantum mechanics [1, 2]. Jiankui He, Qing Chen, Lei Ding, Shao-Long Wan have been investigated the quantum state transfer in randomly coupled spin chains [3]. By using local memories storing the information and dividing the process into transfer and decoding, conclusive transfer is spontaneously achieved with just one single spin chain. X.Q. Xi, J.B. Gong, T. Zhang, R.H. Yue, and W.M. Liu [4] have investigated simple and explicit designs of short isotropic XY spin chains for perfect quantum state transfer are obtained analytically.

Jing-Ling Chen and Qing-Liang Wang [5] have shown that perfect state transfer over arbitrary distances is possible for a simple unmodulated spin chain by some schemes. The transfer of a single qubit state has been investigated in detail by Christandl et al. [Phys. Rev. Lett. 92, 187902(2004)] through a modified Heisenberg XX model Hamiltonian  $H_G$ . The previous study of Christandl was restricted to the excitation states of  $H_G$  (i.e., which correspond to the second subspace of the Hilbert space of  $H_G$ ). They extended their study to the case of the high-excitation states, and the entangled states in the form,  $|\psi\rangle = \alpha|00\dots0\rangle + \beta|11\dots1\rangle$  can be perfectly transferred on the spin chain.

In quantum information processing, transferring a quantum state from one qubit to another is necessary in many cases, e.g., quantum key distribution [6] quantum teleportation [7], and quantum computation [8]. For long-distance quantum communication, there seems no doubt that photons should be the information carriers. For short-distance quantum communication and especially in a solid-state environment, much of the ongoing research efforts are being devoted to spin chains as promising quantum wires. Experimental studies of short spin chains using nuclear magnetic resonances have also emerged [9, 10] Bose first proposed to use a spin chain as a quantum channel for quantum state transfer between different qubits located on two ends of a spin chain [11]. Bose's scheme has motivated many studies focusing on the possibility of perfect quantum state transfer in spin systems [12- 26], i.e., transferring a quantum state with a fidelity one.

In this work, we use the Hamiltonian of a modified Dzyaloshinskii-Moriya model

and investigate the perfect transfer of the quantum state on the spin networks. In this paper, we calculate fidelity in which depends on magnetic field, Dzyaloshinskii-Moriya interaction and another parameters. Then, by using the numerical analysis we show that the perfect transfer of the quantum state is possible with condition  $B \gg \Gamma^2 \omega^{N/2}$ .

## 2 Perfect quantum state transfer

Let us consider the Hamiltonian of a modified Dzyaloshinskii-Moriya model

$$H = - \sum_{j=1}^N (J(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) + D(\sigma_j^x \sigma_{j+1}^y - \sigma_j^y \sigma_{j+1}^x) + \frac{B}{2}(\sigma_j^z + 1)), \quad (1)$$

where  $J$  is coupling strength between lattices  $j$  and  $j+1$ ,  $D$  is the  $z$  component of the Dzyaloshinskii- Moriya interaction,  $\sigma^{x,y,z}$  are the Pauli matrices,  $N$  is the number of sites and  $B$  is the strength of the external magnetic field on every site.

Now we define the raising and lowering operators as  $\sigma^+ = \sigma^x + i\sigma^y$  and  $\sigma^- = \sigma^x - i\sigma^y$ . Therefore, the Hamiltonian(1) can be transformed into

$$H = - \sum_{j=1}^N (\Gamma_{j,j+1} \sigma_j^+ \sigma_{j+1}^- + \Gamma_{j,j+1}^* \sigma_j^- \sigma_{j+1}^+ + \frac{B}{2}(\sigma_j^z + 1)) \quad (2)$$

where  $\Gamma_{j,j+1} = ((J+iD)/2)$ . The Hamiltonian  $H$  obviously describes a nearest-neighbor interaction spin chain. Hamiltonian has  $2^N$  complete and orthogonal eigenvectors, which produce the Hilbert space of spin chain  $\mathcal{H}$ . The Hilbert space of  $\mathcal{H}$  can be divided into  $N+1$  subspaces based on the population of reversed spin [1]. The first subspace has only one eigenvector with zero-value eigenvalue, i.e.

$$|\psi_0\rangle = |00\dots 0\rangle, \quad H|\psi_0\rangle = E_0|\psi_0\rangle, \quad E_0 = 0, \quad (3)$$

where we have denoted  $|0\rangle$  as the state of spin-down  $|-\rangle$ , and  $|1\rangle$  as the state of spin-up  $|+\rangle$ . The ground state  $|\psi_0\rangle$  is a state with all spin down. The first- excitation states, contains  $N$  states, which have the following forms

$$|\psi_1\rangle^{(k)} = \sum_{m=1}^N a_k(m) \phi(m), \quad H|\psi_1\rangle^{(k)} = E_1^{(k)}|\psi_1\rangle^{(k)}, \quad k = 1, 2, \dots, N, \quad (4)$$

where

$$\phi(m) = |00\dots 1_m \dots 0\rangle. \quad (5)$$

Representing Hamiltonian in  $\phi(m)$  basis, we have

$$H = - \begin{pmatrix} B & \Gamma & 0 & 0 & \dots & 0 & 0 \\ \Gamma^* & B & \Gamma & 0 & \dots & 0 & 0 \\ 0 & \Gamma^* & B & \Gamma & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & & \\ 0 & 0 & 0 & 0 & \dots & \Gamma^* & B \end{pmatrix}_{N \times N}, \quad (6)$$

where  $\Gamma = \Gamma_{j,j+1}$  and  $B \gg \Gamma^2 \omega^{N/2}$ . The eigenvalues and eigenvectors for the above Hamiltonian  $H$  obtained as

$$E(\theta) = -(B + 2\sqrt{\Gamma^* \Gamma} \cos(\theta)) ; \quad -\pi \leq \theta \leq \pi, \quad (7)$$

and

$$|\theta\rangle = \beta \begin{pmatrix} \sin(\theta) \\ \omega^{1/2} \sin(2\theta) \\ \omega \sin(3\theta) \\ \omega^{3/2} \sin(4\theta) \\ \omega^2 \sin(5\theta) \\ \vdots \\ \omega^{(N-1)/2} \sin(N\theta) + \frac{\Gamma^*}{B} \omega^{(N-2)/2} \sin((N+1)\theta) \end{pmatrix}_{N \times 1}, \quad (8)$$

respectively, where  $\omega = \Gamma^*/\Gamma$  and  $\beta$  is normalization coefficient which is obtained from

$$\beta^2 = 1 / \{(\omega^{(N-1)}(\sin(N\theta) + \frac{\sqrt{\Gamma^* \Gamma}}{B} \sin((N+1)\theta))^2 + \sum_{n=1}^{N-1} \omega^{n-1} \sin^2(n\theta))\}. \quad (9)$$

We assume that the sender, Alice, has full access to the first qubit  $A$  in the state  $a|0\rangle + b|1\rangle$  and that the receiver, Bab, has full access to the last qubit  $B$  of chain. Then, the state of chain is

$$a|00\dots0\rangle + b|10\dots0\rangle = a|\underline{0}\rangle + b|\underline{1}\rangle, \quad (10)$$

where  $|\underline{n}\rangle$ , corresponding to  $|00\dots1_m\dots0\rangle$ . The coefficient of  $a$  does not change with time as  $|\underline{0}\rangle$  is eigenstate of  $H$  with eigenvalue zero. Therefore, the state  $|\underline{1}\rangle$  will evolve into the superposition of the states with one exactly spin up and all other down. The time evolution of the initial state is

$$a|0\rangle + b|1\rangle \rightarrow a|0\rangle + \sum_{n=1}^N b_n(t)|\underline{n}\rangle. \quad (11)$$

To see how much the transferred quantum state is similar to the original quantum state, a quantity called fidelity is introduced. According to the definition, the magnitude of the fidelity is a real number between 0 and 1. When it is 0, both states are completely different, meaning that the original information is completely destroyed during the transmission process. The value 1 for the magnitude of the fidelity in the process shows that the perfect transfer occurs and original information is completely transmitted.

For the above-mentioned system the fidelity is  $F(t) = \langle \underline{N} | e^{-i\alpha H t} | \underline{1} \rangle = \langle \underline{N} | U e^{-i\alpha H_{\text{diag}} t} U^\dagger | \underline{1} \rangle$ , where  $|\underline{1}\rangle = |1000\dots0\rangle$  and  $|\underline{N}\rangle = |000\dots01\rangle$  and matrix  $U$  is a unitary matrix which makes Hamiltonian  $H$  diagonalized through a similarity transformation. Regarding Eqs.(4) and (8) one can see that  $|\psi_1^{(k)}\rangle = |\theta\rangle$ ,  $a_\theta(1) = \beta \sin(\theta)$ ,  $a_\theta(2) = (\omega)^{1/2} \beta \sin(2\theta)$ , ... and  $a_\theta(N) = \beta((\omega)^{(N-1)/2} \sin(N\theta) + \frac{\Gamma^*}{B}(\omega)^{(N-2)/2} \sin((N+1)\theta))$ . Therefore, the fidelity for the first-excitation states is

$$F(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta a_\theta^*(1) a_\theta(N) e^{-i\alpha t E(\theta)}, \quad (12)$$

where  $E(\theta)$ ,  $a_\theta(1)$  and  $a_\theta(N)$  were defined above. By substituting, one obtain

$$F(t) = (\omega)^{(N-1)/2} \frac{e^{i\alpha B t}}{2\pi} \times \int_{-\pi}^{\pi} \beta^2 \sin(\theta) (\sin(N\theta) + \frac{\sqrt{\Gamma^* \Gamma}}{B} \sin((N+1)\theta)) e^{2i\alpha t \sqrt{\Gamma^* \Gamma} \cos(\theta)} d\theta. \quad (13)$$

To see the quality of transmitting the information for large  $N$  (long distance) in the system, we calculate the fidelity when  $N \gg 1$ . The result is illustrated in Fig.1 which it show fidelity as a functions of  $t$  and  $D$  which it is  $0 \leq F \leq 1$ . Also, Fig.2 show the changes of the fidelity in the time  $t$  for constant other parameters.

### 3 Conclusion

In this paper, we have studied the prefect transfer quantum states on spin chain. In this case, we address the problem of arranging  $N$  interacting qubit, in presence of magnetic field and Dzyaloshinskii-Moriya interaction, in a network which allows the prefect transfer of any quantum state over the longest possible distance. Then, we shown that the prefect quantum state transfer accomplish with condition  $B \gg \Gamma^2 \omega^{N/2}$ .

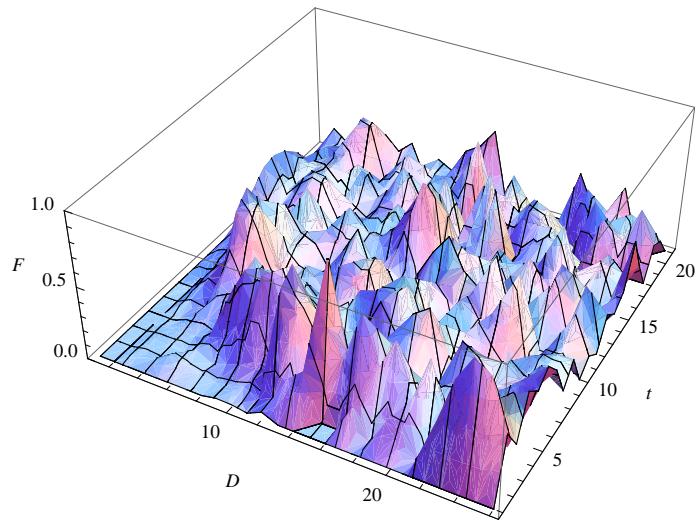


Figure 1:  $F$  as a function of  $t$  and  $D$  where ,  $B = 500$ ,  $\alpha = 1$ ,  $J = 1$ ,  $N = 150$ .

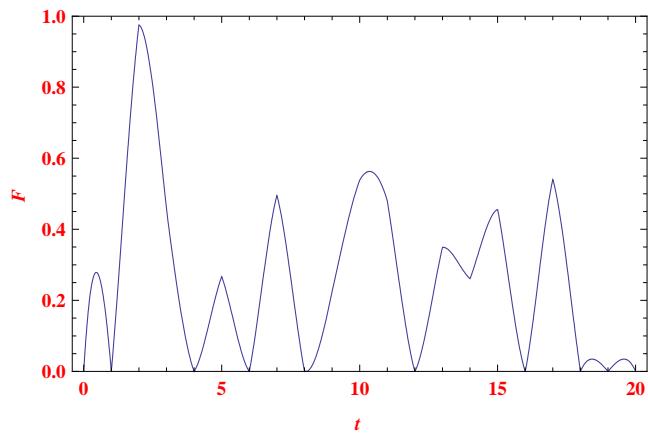


Figure 2:  $F$  as a function of  $t$  where ,  $B = 500$ ,  $\alpha = 1$ ,  $D = 14.455$ ,  $J = 1$ ,  $N = 150$ .

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